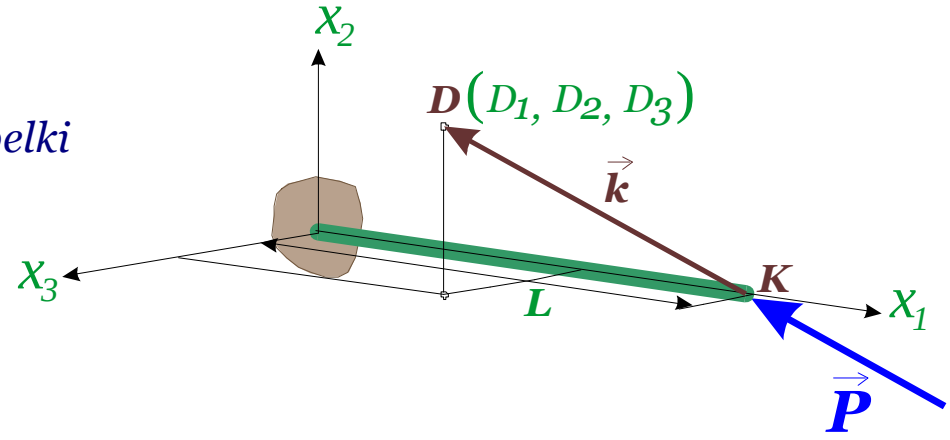
Grupa 2

Zad 1

$L := 7m \qquad P0 := 5kN \qquad b := 12cm \qquad h := 18cm \qquad g := 2cm$

$$D := \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} m$$ - *współrzędne punktu przez który przechodzi kierunek siły*

$$K := \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$$ - *współrzędne punktu K, obciążonego końca belki*



$k := D - K$ - *wektor kierunkowy siły* $\qquad k = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix} m$

$Lk := \sqrt{(k_1)^2 + (k_2)^2 + (k_3)^2} = 6\,m$ - *moduł (długość) wektora kierunkowego*

$$c := \frac{1}{Lk} \cdot k = \begin{pmatrix} -0.666667 \\ 0.666667 \\ -0.333333 \end{pmatrix}$$ - *kosinusy kierunkowe wektora siły P*

$P := P0 \cdot c$ - *składowe wektora siły* $P = \begin{pmatrix} -3.333 \\ 3.333 \\ -1.667 \end{pmatrix} \cdot kN$

$N := P_1 \qquad T2 := P_2 \qquad T3 := P_3$

$N = -3.33333 \cdot kN \qquad T2 = 3.33333 \cdot kN \qquad T3 = -1.66667 \cdot kN$

$M2 := -T3 \cdot L \qquad M3 := T2 \cdot L$

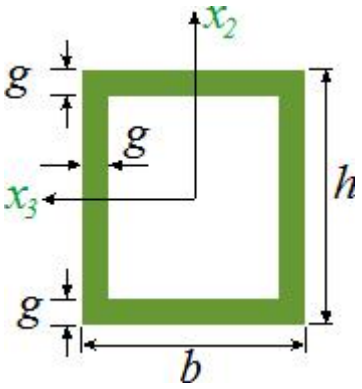
$M2 = 1.16667 \times 10^{1} \cdot kN \cdot m \qquad M3 = 2.33333 \times 10^{1} \cdot kN \cdot m$

$h1 := h - 2g \qquad b1 := b - 2g$

$A := h \cdot b - h1 \cdot b1 = 104 \cdot cm^2$

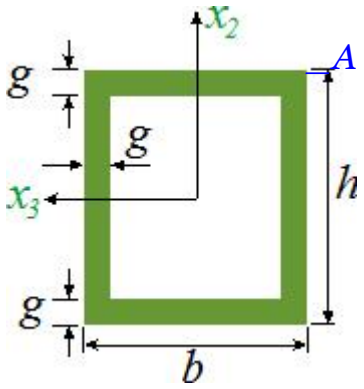
$$J3 := \frac{b \cdot h^3}{12} - \frac{b1 \cdot h1^3}{12} = 4.00267 \times 10^{3} \cdot cm^4$$

$$J2 := \frac{h \cdot b^3}{12} - \frac{h1 \cdot b1^3}{12} = 1.99467 \times 10^{3} \cdot cm^4$$



Naprężenia w punkcie A

$y := \frac{h}{2}$ $z := \frac{-b}{2}$ $a2 := h$ $a3 := b$



$S3 := 0$ $S2 := 0$

$$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = -87.879 \cdot MPa$$

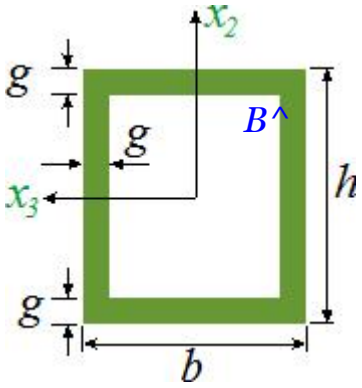
$$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 0.000 \times 10^{0} \cdot MPa$$

$$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = 0.000 \times 10^{0} \cdot MPa$$

$$\sigma_{HMH} := \sqrt{\sigma 11^{2} + 3 \cdot \left(\tau 12^{2} + \tau 13^{2}\right)} = 87.879 \cdot MPa$$

Naprężenia w punkcie B

$y := \frac{h1}{2}$ $z := \frac{-b1}{2}$ $a2 := 2g$ $a3 := 2g$



$S3 := b \cdot g \cdot \frac{(h - g)}{2}$ $S3 = 192 \cdot cm^3$

$S2 := h \cdot g \cdot \frac{(b - g)}{2}$ $S2 = 180 \cdot cm^3$

$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = -64.522 \cdot MPa$

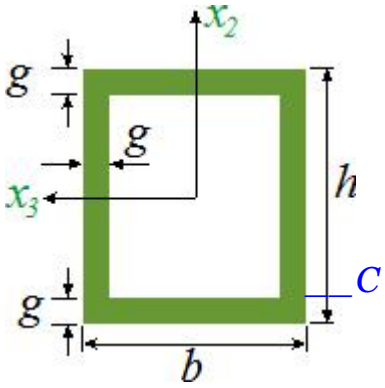
$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 3.997 \times 10^{-1} \cdot MPa$

$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = -3.760 \times 10^{-1} \cdot MPa$

$\sigma_{HMH} := \sqrt{\sigma 11^2 + 3 \cdot (\tau 12^2 + \tau 13^2)} = 64.529 \cdot MPa$

Naprężenia w punkcie C

$y := \frac{-h1}{2}$ $z := \frac{-b}{2}$ $a2 := h$ $a3 := 2g$



$S3 := b \cdot g \cdot \frac{(h - g)}{2}$ $S3 = 192 \cdot cm^3$

$S2 := 0$ $S2 = 0 \cdot cm^3$

$\sigma 11 := \frac{N}{A} - \frac{M3 \cdot y}{J3} + \frac{M2 \cdot z}{J2} = 5.392 \cdot MPa$

$\tau 12 := \frac{T2 \cdot S3}{a3 \cdot J3} = 3.997 \times 10^{-1} \cdot MPa$

$\tau 13 := \frac{T3 \cdot S2}{a2 \cdot J2} = 0.000 \times 10^{0} \cdot MPa$

$\sigma_{HMH} := \sqrt{\sigma 11^2 + 3 \cdot (\tau 12^2 + \tau 13^2)} = 5.436 \cdot MPa$

Zad. 2

$E := 12GPa$ $L := 6m$ $b := 15cm$ $h := 17cm$ $g := 4cm$ $\mu 0 := 0.699156$

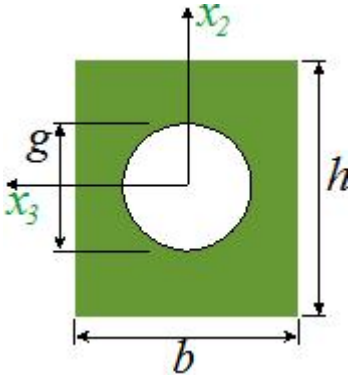
$\mu := 1$ $Lw := \mu \cdot L$

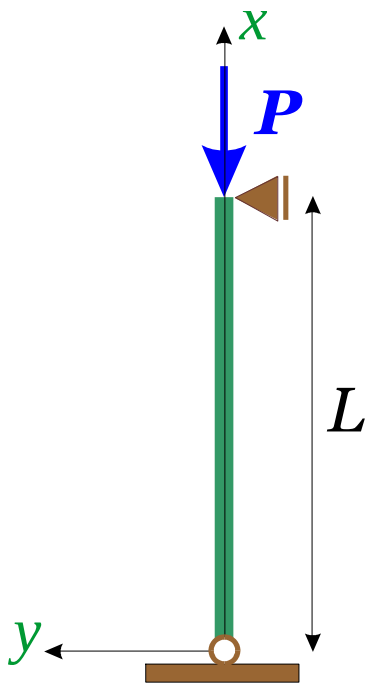
$$J2 := \frac{h \cdot b^3}{12} - \frac{\pi g^4}{64} = 4.768684 \times 10^3 \cdot cm^4$$

$$J3 := \frac{h^3 \cdot b}{12} - \frac{\pi g^4}{64} = 6.128684 \times 10^3 \cdot cm^4$$

$$J := \min(J2, J3) = 4.769 \times 10^{-5}\, m^4$$

$$Pkr := \frac{\pi^2 E \cdot J}{Lw^2} = 156.883 \cdot kN$$

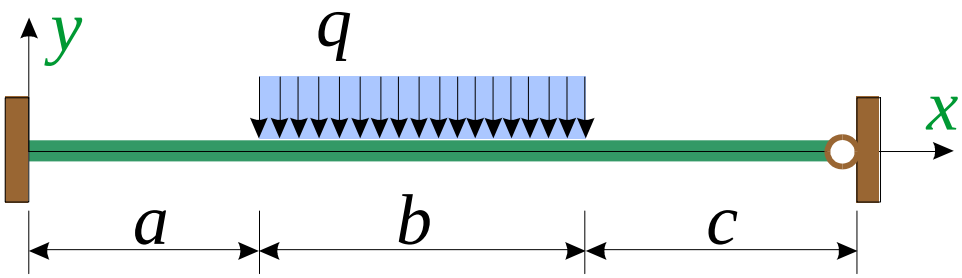




Zad. 3

$q := 5\,\frac{kN}{m}$ $\quad a := 4m$ $\quad b := 4m$ $\quad c := 3m$ $\quad L := a + b + c$

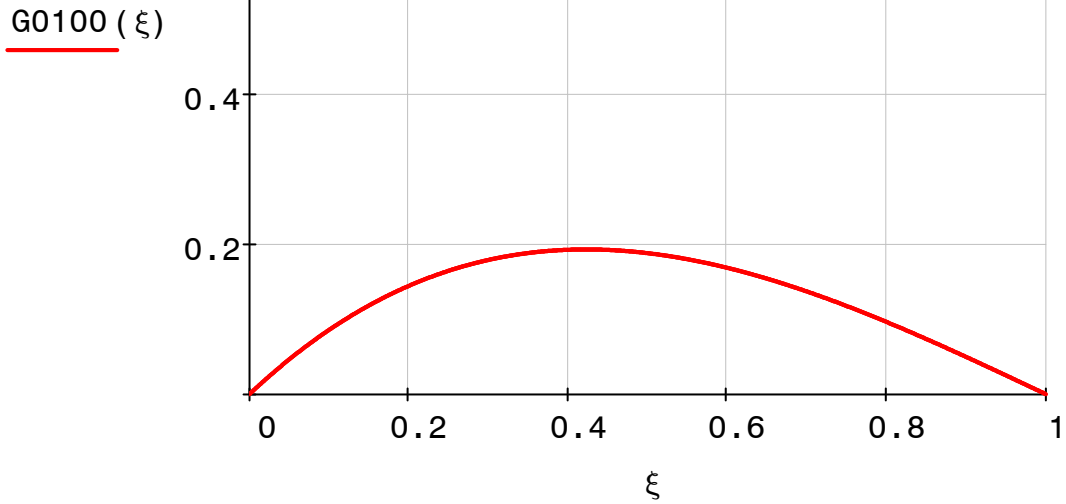
$\xi 1 := \frac{a}{L} = 0.363636$ $\quad \xi 2 := \frac{a + b}{L} = 0.727273$ $\quad G0100(\xi) := \frac{\xi}{2} \cdot \left(2 - 3 \cdot \xi + \xi^2\right)$



Równanie pracy wirtualnej

$M \cdot 1 - q \cdot L^2 \cdot \int_{\xi 1}^{\xi 2} G0100(\xi)\, d\xi = 0$

$M := q \cdot L^2 \cdot \int_{\xi 1}^{\xi 2} G0100(\xi)\, d\xi = 38.016529\,m \cdot kN$



Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

$$H1000(\xi) := 1 - 3\cdot\xi^2 + 2\cdot\xi^3 \qquad H0010(\xi) := \xi^2\cdot(3 - 2\cdot\xi)$$

$$H0100(\xi) := \xi\cdot\left(1 - 2\cdot\xi + \xi^2\right) \qquad H0001(\xi) := -\xi^2\cdot(1 - \xi)$$

Definicja wielomianów Hermite'a dla belki zamocowanej przegubowo:

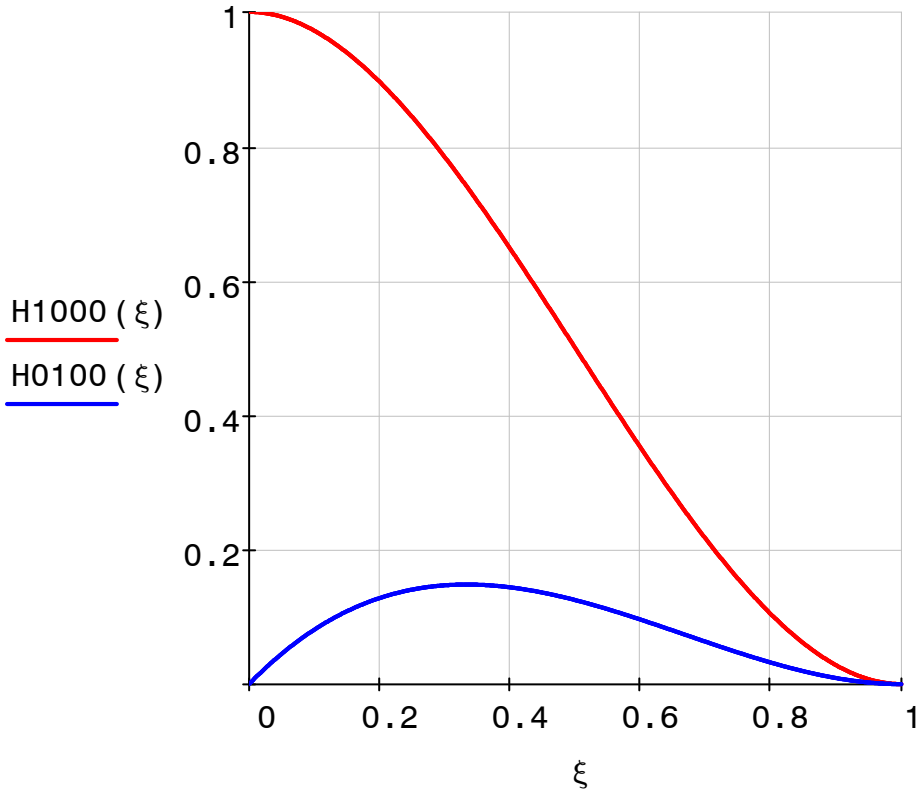
na prawej podporze - G(ξ) *na lewej podporze - K(ξ)*

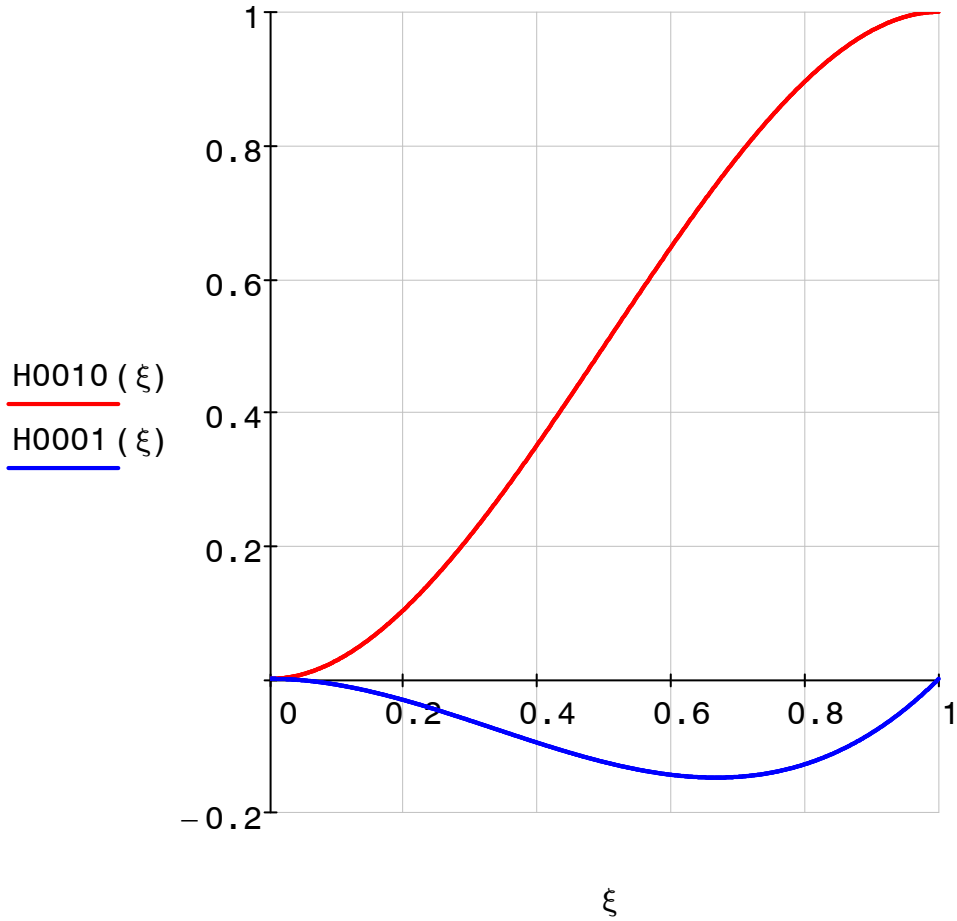
$$G1000(\xi) := 1 - \frac{3}{2}\cdot\xi^2 + \frac{1}{2}\cdot\xi^3 \qquad K1000(\xi) := \frac{1}{2}\xi^3 - \frac{3}{2}\xi + 1$$

$$G0100(\xi) := \frac{\xi}{2}\cdot\left(2 - 3\cdot\xi + \xi^2\right) \qquad K0010(\xi) := \frac{\xi}{2}\cdot\left(3 - \xi^2\right)$$

$$G0010(\xi) := \frac{\xi^2}{2}\cdot(3 - \xi) \qquad K0001(\xi) := \frac{\xi}{2}\cdot\left(\xi^2 - 1\right)$$

Wykresy wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej





Wykresy wielomianów Hermite'a dla belki zamocowanej przegubowo:
na prawej podporze - G(ξ) *lewej podporze - K(ξ)*